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Review Report

on the PhD thesis entitled
"***Axisymmetric solutions of Einstein's field equations with quadrupole***",

by Aray Muratkhan
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The thesis consists from introduction, three chapters, conclusions and the list of references, effectively summarizing relevant research as identified by the author. The work demonstrates the author's solid understanding of theory and good orientation within the subject matter. It contains extensive calculations and a broad array of sources. The text is well-written and well-formatted stylistically to enhance readability and navigability.

In the first chapter, the author presents axisymmetric vacuum solutions to Einstein's equations, with a focus on the quadrupole moment's role. In particular, the author considers static solution with non-zero quadrupole moment and discusses its properties. The chapter provides pedagogical description of the method used for obtaining the solution. The chapter further delves into the dynamics of test particles within these particular spacetimes, both on equatorial and non-equatorial planes.

The second chapter is devoted to the general properties of naked singularities with quadrupole charge and related field equations. It starts from formulation of the cosmic censorship conjecture and continues with addressing the collapse of massive stars into highly dense entities, such as black holes and naked singularities. Further, the author examines axisymmetric vacuum solutions of Einstein's field equations, accounting for gravitational fields surrounding masses with small deviations from spherical symmetry. This small deviation, defined by a quadrupole parameter, results in a curvature singularity not hidden by a horizon, hence a naked singularity. The analysis continues to explore how the quadrupole parameter affects the geodesics, noting differences with respect to the properties observed in Schwarzschild spacetimes.

The third chapter explores the astrophysical shadows cast by static quadrupole naked singularities, showing that these shadows display distinct features, particularly regarding their size, which diverges from the Schwarzschild spacetime case. This variation highlights the quadrupole parameter's impact on shadow sizes in q-metric spacetimes and suggests the potential observability of quantum gravity effects near naked singularities. The study compares the gravitational effects of black holes and naked singularities, indicating that, although their influences are broadly similar, specific physical parameters may distinguish them.

The significance of using Einstein's theory of relativity to understand gravitational interactions around large celestial objects, such as stars and black holes, is emphasized. This

research introduces a novel approach to investigating naked singularities by examining mass distribution, especially in cases deviating from perfect sphericity, marked by a quadrupole moment.

The work is theoretically sound, supported by an adequate literature review, includes the necessary mathematical framework, and comprises several interconnected chapters. It details various tasks that well describes the discussed topics, the outcomes of these tasks, and scientific findings based on the results.

In conclusion, Murathan Arai's dissertation titled "Axisymmetric Solutions of Einstein's Field Equations with Quadrupole" fulfills all criteria for a Doctor of Philosophy (PhD) degree. I recommend to the Scientific Council the acceptance of Arai's defense for the degree of Doctor of Philosophy (PhD) in Physics and Astronomy.

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